Hybrid Back-Projection: Theory and Future Direction of Seismic Array Analysis



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Collaborators:

FUKAHATA Yukitoshi (DPRI, Kyoto University) KASAHARA Amato, NAKAO Atsushi, EZAKI Hayaki (Univ. of Tsukuba) I) Roughly continuous data in space and time is obtained by seismic array.

2) We have inversion method for only low sampling data & Back-Projection

We want to get "New results" not only using "New data" but also with "New formulation"!

Array data

Facts: Array data contains similar signal; similar modeling error.

Back-projection:

Stacking > similar signal > seismic source image (?) **Problem of Back-projection:**

BP will project not only signal but also correlated error **Better solution: Hybrid Back-projection**

Mitigating correlated error effect using Green's function.

Other Solutions:

- Introducing the data covariance matrix into seismic source inversion.
- Designing array to mitigate the effect of modeling error in Back-projection (Station coverage is important!)

Data Covariance Matrix

Yagi & Fukahata (2011, GJI) Solution: Introducing "Uncertainty of a Green's function" into waveform inversion.

As a result: high resolution result with time (about five times higher resolution)



But They neglected the space correlation!

What is to be done? **My Answer**: we should construct new formulation for array data.

Theoretical Background of BP

By Fukahata; Yagi; Rivera (2012, submitted to GJI)

Basic Equation of the BP

$$S_{l}(t) = \sum_{j} \alpha_{j} \dot{u}_{j} \left(t + t_{lj}^{P} \right)$$

$$= \sum_{j} \alpha_{j} \sum_{i} \left(\ddot{D}_{i} * G_{ij} \right) \left(t + t_{lj}^{P} \right)$$

 $\begin{aligned} & \mathcal{U}_{j} : \text{displacement for station "}j'' \\ & t_{lj}^{P} : \text{Travel time from source} \\ & \text{grid "}l'' \text{ to station "}j''' \\ & \boldsymbol{\alpha}_{j} : \text{Normalizing factor} \end{aligned}$

 D_l : Slip function on grid "l" G_{lj} : Green's function from grid "l" to station "j"

Important assumption in BP: Waveform due to slips on the grid except for the grid "*l*" are cancelled out earth other (Ishii et al., 2005).

$$S_{l}(t) \approx \sum_{j} \alpha_{j} \left(\ddot{D}_{l} \ast G_{lj} \right) \left(t + t_{lj}^{P} \right) \quad \text{or} \quad \approx \sum_{j} \alpha_{j} \left(\dot{D}_{l} \ast \dot{G}_{lj} \right) \left(t + t_{lj}^{P} \right)$$

Theoretical Background of BP

By Fukahata; Yagi; Rivera (2012, submitted to GJI)

Implicit Assumption in BP: Green's function is assumed to be like the delta function, since we only use predicted travel time in BP.

$$S_l(t) \sim \sum_j \alpha_j \ddot{D}_l(t)$$
 or $\sim \sum_j \alpha_j \dot{D}_l(t)$

BP is directly related not to seismic energy release but to slip-acceleration (or slip-rate) on fault plane!

BP well work for deep earthquake



By Suzuki & Yagi (2011, GRL)

Green's function can be assumed to be like the delta function.

Rupture velocity



Hybrid Back-Projection (HBP)

By Yagi; Nakao; Kasahara (2012, EPSL)

Problem: P-waveform trains contains pP and sP phases; Green's function can not be approximated to Delta function.



Solution:

Cross-correlation of an observed waveform with the the theoretical Green's function

Theoretical Background of HBP

By Fukahata; Yagi; Rivera (2012, submitted to GJI)

Basic Equation of the HBP $S_{l}(t) = \sum_{j} \alpha_{j} (\dot{u}_{j} \bigstar \dot{G}_{lj})(t)$ $= \sum_{j} \alpha_{j} \sum_{i} \left[(\dot{D}_{i} \ast \dot{G}_{ij}) \bigstar \dot{G}_{lj} \right](t)$

 $\boldsymbol{\mathcal{U}}_{j}$: displacement for station "j"

 α_i : Normalizing factor

 D_l : Slip function on grid "l" G_{lj} : Green's function from grid "l" to station "j"

Important assumption in HBP: Waveform due to slips on the grid except for the grid "l" are cancelled out earth other.

$$S_l(t) \approx \dot{D}_l(t) * \sum_{j \ lj} (\dot{G}_{lj} \bigstar \dot{G}_{lj})(t)$$

Theoretical Background of HBP

By Fukahata; Yagi; Rivera (2012, submitted to GJI)

Assumption in HBP: Auto correlation of greens function is assumed to be the delta function.

$$S_l(t) \sim \sum_j \alpha_j \dot{D}_l(t)$$

Results of HBP is directly related to slip-rte on fault plane!

BP vs HBP (Impulse response)



Green's function of shallow earthquake can not be approximated to Delta function.

Synthetic Test





(b) Hybrid back-projection image



F2 (0.5 - 2.0 Hz)

F2 (0.5 - 2.0 Hz)



F1 (0.1 - 0.5 Hz)





F1 (0.1 - 0.5 Hz)

F2 (0.5 - 2.0 Hz)





The conventional BP method produced a more blurred image than the hybrid method for both frequency bands.

Application to Real Data



The large slip-rate on the shallow part of the fault along the Japan Trench was evident only for the Low frequency dataset.

HBP and Inversion



F2 (0.5 - 2.0 Hz)



The shallow event corresponds to the rapid and smooth acceleration of the slip-rate function near the trench.

> Source model from Yagi and Fukahata (2011, GRL)



Conclusion

- Array data contains signal and modeling error.
- The advantages of the BP method are much less computation and no necessity of Green's function.
- BP will project not only signal but also modeling error.
- HBP has enabled us to estimate detailed motion of rupture front and frequency dependence.
- One the other hand, we should verify image of HBP by using result of seismic source inversion.
- To get reasonable HBP image, we should develop....(e.g. empirical Green's function for high frequency)
- Results of the suitable seismic source inversion have useful information in frequency characteristic and motion of rupture front.
- Inversion with data covariance should be effective way for the array data analysis. > [first priority]